change from westerly to southerly is contrary to what might be expected. However, the explanation is to be found in the fact that precipitation occurred first to the south, and as the gradient changed to cause south-southwest wind, cloudiness and lower temperature were

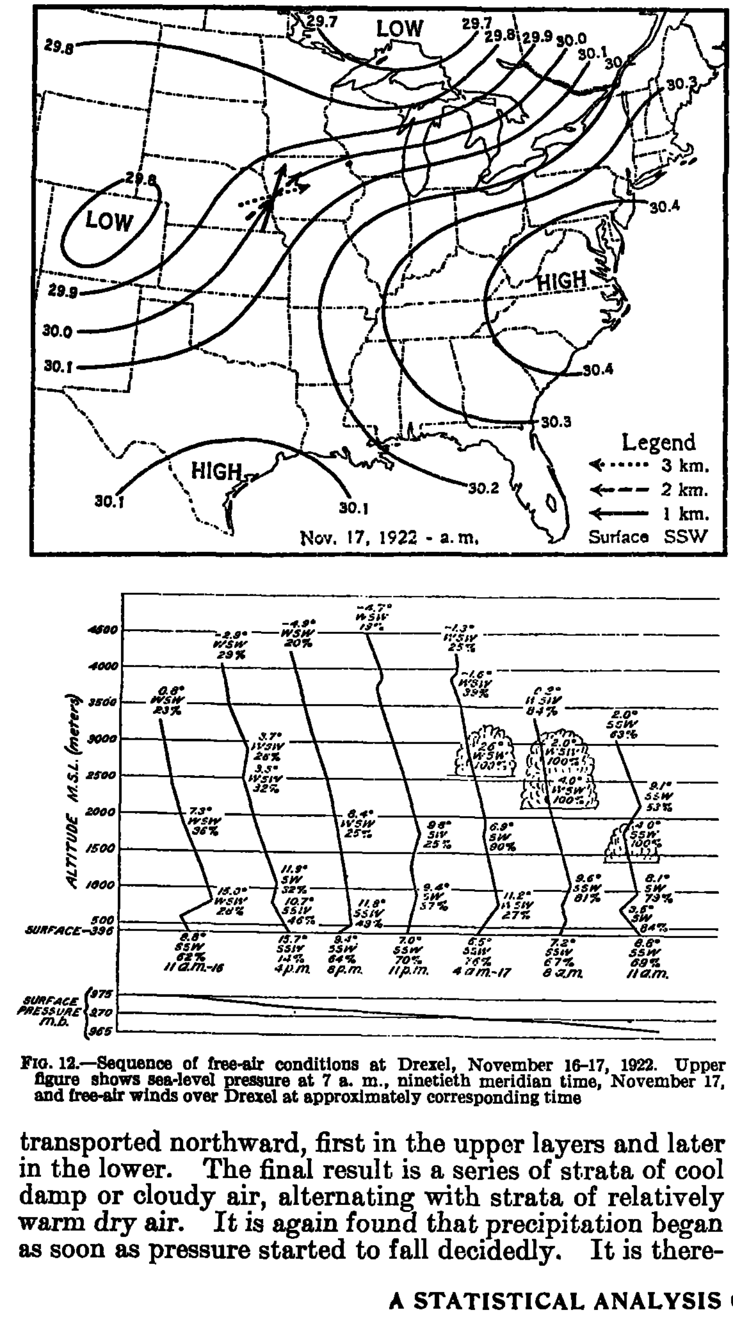

Fig. 12.—Sequence of free-air conditions at Drexel, November 16-17, 1922. Upper figure shows sea-level pressure at 7 a. m., ninetieth meridian time, November 17, and free-air winds over Drexel at approximately corresponding time

transported northward, first in the upper layers and later in the lower. The final result is a series of strata of cool damp or cloudy air, alternating with strata of relatively warm dry air. It is again found that precipitation began as soon as pressure started to fall decidedly. It is therefore probable that the mere transport of air from regions where precipitation is occurring can not of itself cause other than very light precipitation over the region affected. Whether such accumulation of clouded air is brought about in a deep layer having an unbroken lapse rate, or in a series of relatively shallow strata consisting of alternate lapses and inversions in temperature, it is apparent that in neither case is precipitation of consequence possible until there occurs the pressure change necessary to bring about forced ascent of air. The case of this series compels the opinion that rain was caused neither by instability, inasmuch as there was no deep layer having strong lapse rate in temperature, nor by ascending a slope of discontinuity, since the wind directions were very nearly uniform with altitude.

CONCLUSIONS

A feature of these graphs is the fact that but few of them show any prominent differences distinguishing them from the rest. This apparent lack of individuality is conspicuous even in cases where pronounced differences in the temperature-altitude curves and other data attached thereto would naturally be inferred. For example, while the graph of November 8-9, 1917, is one of the most striking of the collection, it is by no means typical of the general pressure situation in which it is classified; in fact it is quite closely imitated by the graph of June 12, 1917, which portrays free-air conditions under a decidedly different situation of surface pressure.

It is obvious that the most important group differences shown by the graphs pertain to seasons rather than to any other divisions, arbitrary or otherwise. The reason for this seasonal influence is easily understood by referring to tables of average free-air data, in which, particularly for continental sections, seasonal peculiarities in the vertical temperature gradients are readily identified. While the division into types is nevertheless justified, the testimony of the graphs seems to be that by whatever processes precipitation develops in a given season, the vertical structure of temperature and humidity shows substantial similarity when the precipitation stage begins.

The conclusion seems well founded that the various processes of precipitation formation ultimately resolve themselves into a free-air structure comprising strata of greater or less depth having lapse rates equal to or greater than the moist adiabatic rates for the prevailing temperatures. Notwithstanding this, consideration of precipitation types in connection with surface and free-air conditions should lead to a better understanding of the subject, to the end that these types and their development may be recognized on the weather maps.

A STATISTICAL ANALYSIS OF SOLAR RADIATION DATA

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SYNOPSIS

An attempt is made to determine by statistical analysis the degree of validity of the values of solar intensity derived from pyrheliometric or bolometric observations. Both correlation coefficients and the mean dispersion of the data are employed. Certain physical relations are assumed to exist between errorless elements of data and constitute criteria for determining the validity of derived values.

Numerous correlation coefficients between both the daily values and the monthly means of solar radiation elements at Mount Wilson are presented. A high negative correlation, averaging -0.55 to -0.60, between the apparent atmospheric transmission coefficient, a and the apparent solar constant, A_0, is a feature of all daily pyrheliometric data. This is known to be due to changing transparency between low and high sun observations. A similar high negative correlation both for daily values and monthly means exists between the solar constant, E'_0, and the transmission coefficient. Since zero correlation should exist between these elements, it is inferred that the derived values, E'_0, are a function of atmospheric transparency.

A high negative correlation, -0.60, between the mean monthly values of E'_0 and A_2, is regarded as strong evidence of the unreality of the variations of the monthly values of E'_0.

A high correlation, +0.87, exists between the yearly means of the scatter of a and of the scatter of E_0, and a negative correlation, −0.74, between the yearly means of A_2 and the scatter of E_0. These high correlations where none should exist show that the scatter of E_0 is due to atmospheric variations.

It is shown that as the elevation increases the scatter of both the observed intensity, A_1, and of the derived value of the solar constant, E_0, decrease *pari passu* with the scatter of a; and the inference is that these variations would almost vanish at the outer limit of the earth's atmosphere.

A selection of 53 high grade days at Mount Wilson, 1.8 km., yields a probable error of ±0.65 per cent for A_1, whereas the probable error of E'_0 for all days is 60 per cent greater.

Five sets of pyrheliometric observations taken by different observers at stations 3.3 to 4.5 km. in elevation give results substantially in agreement, in indicating that at 4.0 km. on high grade days with vapor pressure averaging 1.5 mm. the probable error of A_1 is about ±0.40 per cent.

Since the probable error of the pyrheliometer reading is about ±0.30 per cent, it is obvious that after allowing for the error of extrapolation to air mass one, and for residual atmospheric variations at an elevation of 4 km. little is left for possible solar variation.

Introduction.—The solar radiation data analyzed and discussed in this paper include the solar constant measurements made by Abbot [1] at Washington, Mount Wilson, Mount Whitney, Bassour, and Calama; and fragmentary pyrheliometer observations made under the direction of Bigelow [2] at Cordoba and La Quiaca, Argentina, and at La Confianza, Bolivia; and by the Ångströms at the Peak of Teneriffe [3] and Mount Whitney.[4]

Following the notation of the Astrophysical Observatory, the symbols representing the elements referred to in the paper are, e, the vapor pressure in millimeters; $p.\ w.$ the precipitable water in the entire atmosphere above the station as determined by the bolometer; a, the apparent atmospheric transmission coefficient, or the coefficient for a definite wave length, as 0.8 μ; A, the intensity of solar radiation at the earth's surface, with subscripts *0*, *1*, *2*, *3*, etc., to indicate the air mass. If the sun be in the zenith, the intensity is A_1; E_0 is the solar constant, or E'_0 with water vapor correction applied. The mathematical relations between the pyrheliometric data are simply expressed by the formulas, $A_1 = A_0\ a$; $A_2 = A_0\ a^2$.

The statistical processes which have been employed in the investigation include the computation of coefficients of correlation for both daily values and monthly means, and the determination of the mean day-to-day variability for each year for all the elements. The method of correlation mainly employed is that described in this REVIEW, volume 52, 1924, page 424. The mean variability, or the mean of the consecutive variations without regard to sign, is regarded as the appropriate index of dispersion for employment in this investigation, because it is wholly independent of any secular or seasonal variations which may occur in one element and not in the other and which would render the standard deviations of the two elements not comparable with each other.

Physical relations between elements of data.—For the interpretation of the statistical coefficients, certain physical relations which should be found between these elements are presented below because they constitute criteria for determining the validity of the derived values available for analysis. Such data comprise all values derived from the pyrheliometric observations of intensity, including a, A_0, A_1, also from the bolograph trace, as E_0, E'_0, $p.\ w.$ and a_λ. The symbols and items of data are assumed to represent real values, unaffected either by errors of direct observation or by the frequently greater errors of derivation.

(1) High positive correlation between a and A_1, A_2, A_3, A_4, the correlation increasing with the air mass.

(2) Zero correlation between a and A_0. The latter is an approximate solar constant, with atmospheric effects supposed to be eliminated by the process of extrapolation to zero air mass.

(3) Zero correlation between a and E_0.

(4) Zero correlation between the dispersions of a and E_0.

(5) Zero correlation between E_0 or its dispersion, and $p.\ w.$, or e.

(6) Positive correlation, increasing with elevation, between the values of E_0 and A_1, also between their dispersions. The magnitude of this correlation should vary with the ratio of solar to total fluctuations.

(7) The dispersion of a should show a progressive decrease with increase in elevation above sea level.

(8) The dispersion of E_0 should not vary with change of elevation, but that for A_1 should decrease appreciably unless solar changes are relatively great.

Correlation of daily data.—There are available 15 years of Mount Wilson data, but the correlation coefficients are computed mainly for the years 1908–1918. The following table shows the coefficients for daily data at Mount Wilson, based on 500 to 800 values.

TABLE 1.—*Correlation coefficients between daily values of solar radiation elements at Mount Wilson*

	E_0	A_0	A_1	A_3	a	a (at 0.8μ)	$p.\ w.$	e
E'_0	+0.92	+0.66	+0.49			−0.61	+0.23	
E_0		+.71	+.59	+0.27		−.57	−.40	−0.42
A_0			+.77	+.42	−0.51		−.52	
A_1				+.74	+.40		−.70	−.51
A_3					+.80		−.73	−.52
a						+.71	−.56	−.40
$p.\ w.$								+.60

The table below shows the coefficients for the daily data at Cordoba for the year 1912, based on 85 values.

TABLE 2.—*Correlation coefficients at Cordoba*

	A_0	A_1	A_2	A_3	A_4
a	−0.57	+0.38	+0.81	+0.88	+0.89
A_0		+.71	+.25	−.23	−.28
A_1			+.69	+.61	+.60
A_2				+.95	+ 95
A_3					+.99

Tested by criterion (1) above, the positive correlations of a with A_2, A_3, A_4, are physically consistent, but the magnitude of the coefficient for A_1 seems too low. The comparatively large negative correlation between a and A_0 is a characteristic of all pyrheliometric data.[5] For example, at Washington it is −0.51; at Bassour, −0.65; Helwan, −0.70; Hump Mountain, −0.70 and at La Quiaca, −0.51. The existence of this high negative correlation between a and A_0 is evidence of atmos-

[1] Annals of the Astrophysical Observatory, Vol. III, 1913; IV, 1922. Washington. By C. G. Abbot, Director.

[2] Bigelow, F. H. The Thermodynamics of the earth's atmosphere from the surface to the vanishing plane. Oficina Meteorológica Argentina. Boletin 4, 1914. Buenos Aires.

[3] Ångström, Knut. Intensité de la radiation solaire à différentes altitudes. Recherches faites à Ténériffe, 1895 et 1896. Upsala, 1900.

[4] Ångström, A. K. and Kennard, E. H. Some pyrheliometric observations on Mount Whitney. Astro. Jour., 39–350. 1914.

pheric control (Criterion 2), and is the result of changing transparency between low and high sun observations.[6]

Referring to Table 1, there is shown a coefficient of -0.61 between E'_0 and the transmission coefficient a at wave length 0.8μ. This is higher even than the correlation between a and A_0. Real values of the solar constant and a should yield zero correlation. *The existence of this high negative correlation between a purely atmospheric datum and alleged daily solar constant values constitutes strong evidence against their reality as solar changes.*

The correlation, $+0.49$, between E'_0 and A_1, at Mount Wilson (Table 1) if considered entirely by itself implies a day-to-day solar variation amounting to about one-third as much as that caused by all other variations. This inference, however, is not consistent with the other correlations in Table 1, and is, moreover, invalidated by all the other findings in this paper.

Correlation of monthly data.—The advocates of day-to-day solar changes may, however, argue that daily values may be adversely affected, and such changes obscured, by uneliminated atmospheric variations, but that real long-period solar changes will appear when the accidental features of the daily values are eliminated by averaging a large number of observations. This view has some plausibility, but when monthly means are correlated, similar close relations, where none should be found, are still in evidence as shown by the high correlation coefficients in Table 3. These coefficients are based on the 75 monthly means available for 15 years at Mount Wilson.

TABLE 3.—*Correlation coefficients between monthly mean values of solar radiation elements at Mount Wilson, 1905–1920*

$n=75$

	E_0	A_0	A_1	A_2	a	a (at $.8\mu$)	$p. w.$	e
E'_0	+0.85	+0.45	−0.48	−0.60	−0.67	−0.64	+0.42	
E_0		+.56	+.12	−.35	−.54	−.66	−.31	
A_0			+.62	+.43	−.33	−.35	−.48	
A_1				+.84	+.62	+.58	[1] −.80	[1] −0.57
A_2					+.79	+.62	[1] −.77	[1] −.52
a						+.83	[1] −.60	[1] −.50
$a. 8\mu$							[1] −.43	[1] −.50
$p. w$								+.46

[1] Exclusive of the years 1912 and 1913, on account of Katmai dust causing abnormally depressed values of a and A_1.

The correlation between E'_0 and a at wave length 0.8μ is -0.64 ± 0.05, as compared with -0.61 for the daily values. As in the case of the latter, this high negative correlation where none should exist, forbids any conclusion that the monthly differences of E'_0 represent, even approximately, real solar changes.

If solar changes occur from month to month, there should be a positive correlation between E'_0 and A_1 (Criterion 6), and less correlation, but still positive, between E'_0 and A_2. These observations over a period of 75 months, however, show a *high negative correlation, -0.60 ± 0.05*, between E'_0 and A_2 which is to be interpreted as additional striking evidence that the fluctuations of even the monthly means of E'_0 at Mount Wilson are caused by atmospheric and terrestrial rather than solar changes. The value A_2 is a purely terrestrial datum obtained by practically direct observation.

The correlation of E_0 with A_2 is less than that of E'_0, while for E_0 and A_1 there is a small positive correlation. This may mean that E_0 is a more representative value of the solar constant than E'_0. It is more probable however that it means simply that the systematic influences tending to cause a high negative correlation of E'_0 with A_2 are somewhat less operative in the case of E_0.

Incidentally, it may be stated that the correlation between the monthly means of E'_0 and the monthly sunspot numbers is -0.29 ± 0.07, certainly very slight evidence of relationship. On the other hand, the correlation of the sunspot numbers with a purely terrestrial datum, A_2, is $+0.56 \pm 0.05$. There is as yet no satisfactory interpretation of this rather large coefficient.

The intensity, A_2, has, as must be expected, a high negative correlation, -0.77, with the precipitable water. Mean monthly values of these elements in Table 4 below show well marked annual periodicities with phases in opposition, i. e. a maximum *p. w.* and a minimum A_2 in July, which is consistent with the high negative correlation. The negative correlation between *p. w.* and a, $-.60$, is also consistent with the opposition in phase, of their annual fluctuation.

TABLE 4.—*Monthly means of solar radiation elements at Mount Wilson, 1905–1920*

Element	May	June	July	Aug.	Sept.	Oct.	Nov.
$p. w.$ (mm)	7.0	8.3	11.7	11.1	9.5	8.4	5.1
[1] A_2 (cal.)		1.398	1.370	1.370	1.393	1.413	1.430
[1] E_0 (cal.)	1.935	1.945	1.946	1.945	1.940	1.939	1.931
[1] a		.889	.887	.887	.894	.900	.910
e (mm)	4.6	5.3	5.9	5.8	5.0	4.4	3.8

[1] Abnormal values in 1912 and 1913 were disregarded in deriving the means for E'_0, A_2, and a.

The high negative correlation, -0.60 ± 0.05, Table 3, between E'_0 and A_2, derived from the month to month changes, should likewise lead us to infer *an annual periodicity in E'_0 opposite to that in A_2*. The mean monthly values confirm this deduction from the correlation coefficients. The positive correlation $+0.42$, between *p. w.* and E'_0, is likewise confirmatory.

The existence of this annual period is further evidence of the unreliability of E'_0 as a measure of solar intensity.

The mean variability of the data at Mount Wilson.—Considering now the other statistical index utilized, there are found similar close relations between alleged solar and purely terrestrial data.

The mean value and the mean variability of a number of solar radiation elements as computed for each year for Mount Wilson are shown in the following Table 5 with the averages for the whole period, excluding 1912, when the Katmai dust caused abnormal yearly means and an excessive scatter of all data. These yearly mean variabilities are derived from day-to-day changes, averaging about 75 per year. This measure of dispersion is independent of any systematic or secular variations occurring in one variant and not in the other; moreover, as already pointed out it is through the day-to-day changes that E_0 and a are so closely related.

[5] More clearly to reveal the fact that the daily values have a high negative correlation the formula employed must exclude the influence of the marked seasonal variations in a, which strongly affect the coefficients obtained by the ordinary method. This is accomplished by the formula used in the present instance. Thus, the Mount Wilson data selected by Abbot in his paper "The temperature and radiation of the sun," (Astrophys. Jour., 1916), to prove the nonexistence of an alleged negative correlation between a and E_0, yield by the ordinary method only -0.06 ± 0.10, an apparently negligible amount. Correlating the same data by the method used in this paper gives a coefficient of -0.58 ± 0.07.

[6] On days when the intensity at a large air mass, A_4 for example, is very low, there is a low real value of a; but with the tendency to increased transparency toward midday, frequently characteristic of such days, a, the apparent transmission as measured by the slope of the line joining the low and high sun observations, is too low and there result too high values of A_0. With a high value of A_4, on the other hand, there is associated a high real value of a, but since there is on such days a tendency toward decreasing transparency, caused by strong convection and increased evaporation, the depressed slope of the line causes too high values of the apparent transmission and too low values of A_0.

TABLE 5.—*Yearly means and yearly mean variabilities of solar radiation data at Mount Wilson*

Yearly mean				Mean variability (±)			
Year	a	e	A_2	a	E_0	A_0	A_1
		mm.	*cal.*		*cal.*	*cal.*	*cal.*
1905	0.896	7.4	1.397	0.018	0.040	0.053	------
1906	.893	6.2	1.395	.013	.033	.053	------
1908	.886	4.8	1.379	.013	.033	.038	0.033
1909	.895	5.3	1.400	.013	.036	.041	.041
1910	.900	5.4	1.408	.012	.036	.043	.039
1911	.902	5.1	1.428	.012	.036	.041	.034
1912	.837	5.0	1.229	.030	.072	.059	.049
1913	.878	4.8	1.314	.016	.044	.047	.040
1914	.892	5.1	1.374	.014	.034	.045	.041
1915	.895	4.4	1.413	.012	.027	.047	.043
1916	.888	5.2	1.382	.015	.030	.044	.033
1917	.881	5.5	1.353	.017	.032	.042	.041
1918	.888	5.6	1.368	.017	.032	.048	.045
1919	.884	4.6	1.360	.014	.024	.043	.045
1920	.886	5.5	1.353	.017	.031	.042	.048
Means	.890	5.6	1.380	.014	.033	.043	.040

Table 5 shows the relative amounts of scatter expressed in calories for A_1, A_0, and E_0. The average yearly mean of A_1 is 1.56 cal.; of A_0 1.74 cal. A part of the scatter of A_1 is due to the error of extrapolation from the high sun observation, the air mass varying from about 1.2 in June to 1.5 in October.

This is shown by the small values of the correlation between A_1 and a, +0.40 at Mount Wilson, in latitude 34°, Table 1, and +0.38 at Cordoba, in latitude 31°, Table 2. At La Quiaca, on the other hand, in latitude 22°, the correlation is +0.50. The smaller coefficients at higher latitudes are very likely due to increased scatter involved in extrapolation from a larger air mass.

Extrapolation increases scatter at air mass zero but reduces scatter due to day-to-day changes in air transparency. The net result, however, as shown by Table 5, is that the scatter of A_0 differs but little from that of A_1. At Mount Wilson the mean variabilities of these values, in percentages, are A_1, ±2.57; A_0, ±2.47. At La Quiaca the variabilities are A_1 ±1.34; A_0, ±1.28. At Cordoba the percentage variability of A for various air masses is as follows: A_4, ±12.3; A_3, ±8.6; A_2, ±5.3; A_1, ±3.7; A_0, ±4.0.

It is clear, therefore, that the error of extrapolation is responsible for a very considerable part of the scatter of A_1, and for a still larger proportion of the scatter of A_0. Since the correlation of A_0 and E_0 is very high, +0.71 (Table 1), it is inferred that the scatter of E_0 is very largely of terrestrial origin.

The yearly means in Table 5 show rather high correlation. Between a and A_2, it is +0.87. (Criterion 1.) Some of the data in this table are plotted in Figure 1. It is clear that the yearly scatter of E_0 varies directly with the scatter of a and inversely with the mean intensity A_2. (The scatter of E'_0 is only 1 per cent less than that of E_0.) The correlation between the data of curves 1 and 2 is +0.87±0.05; between curves 1 and 3 it is −0.74±0.07, the latter curve being inverted. (These correlations were computed by the ordinary method, there being only 15 values.) It will be seen that in 1912, the year of the Katmai eruption, and to a less extent in 1913, the scatter of E_0 and a was abnormally increased and the intensity A_2 decreased. (Cf. Criteria 4 and 6.)

In curve 1 a dotted line drawn from 1911 to 1914 shows the probable course in the intervening years if the Katmai dust had not prevailed. Thus adjusted, curve 1 shows a high correlation with curve 4, the mean vapor pressure. Theoretically, the correlation between these elements should be zero. (Criterion 5.)

Here again, the yearly data, based on more than 1,000 daily values, show high coefficients where zero correlation should exist, and confirm the previous inference from the monthly means and the daily data that *a very large part, if not all, of the variations in E_0 must be ascribed to atmospheric causes.*

Smallness of possible solar variation shown by progressive decrease in scatter of all data with increasing altitude.—If solar variations are a large part of the day-to-day changes in E_0, or derived values of the solar constant, and the errors are small, then the higher in the atmosphere the daily observations are made, the more must the errors diminish and the variations of E_0 become relatively greater. What do the data show?

There are available for this purpose observations at the following stations: Washington, 39° N.; Mount Wilson, 34° N.; Mount Whitney, 37° N.; Teneriffe, 28° N.; Bassour, 36° N.; Calama, 22° S.; La Quiaca, 22° S.; La Confianza, 22° S.; Cordoba, 31° S. Washington and Cordoba are low-level stations having somewhat similar characteristics. Mount Wilson is a peak well within the limit of the lower convection currents. Mount Whitney has an elevation above the lower convection layer, insuring great steadiness of atmospheric conditions except for local ascending currents near midday in the vicinity of the peak. La Quiaca and La Confianza are on a high plateau in a dry region.

The mean variability [7] of various elements for each of the stations is given in Table 6, and plots of the percentage variability for a and A_1 are shown in Plate 1, Figure 2, and for a and E_0 in Figure 3.

TABLE 6.—*Mean value and mean variability of solar radiation elements at stations with elevations ranging from sea level to 4.5 kilometers.*

Years	Stations	Elevation, meters	Mean value			Mean variability (±)					
			A_1	e	a	a	A_1	A_0	E_0	Per cent a	Per cent A_1
			cal.	*mm.*			*cal.*	*cal.*	*cal.*		
1905–1907	Washington	10	1.364	6.0	0.818	0.051	0.098	0.112	0.115	6.5	6.8
1912–13	Cordoba	450	1.440	9.0	.828	.031	.052	.067	------	3.8	3.7
1911	Bassour	1,160	1.456	7.8	.856	.027	.042	.045	.060	3.3	2.9
1912	do	------	1.340	7.2	.801	.041	.069	.083	.034	5.1	5.1
1908–1920	Mount Wilson.	1,750	1.560	5.0	.890	.014	.039	.044	.034	1.6	2.5
1912	do	------	1.494	5.0	.844	.030	.049	.059	.070	3.6	3.3
1918–19	Calama	2,200	1.590	6.0	.902	.010	.035	.033	.026	1.2	2.2
1913	La Quiaca	3,500	1.700	1.5	.903	.008	.023	.024	------	0.9	1.4
Do	Mount Whitney.	4,420	1.700	2.5	.930	.006	.015	------	------	0.6	0.9
Do	La Confianza.	4,460	1.740	1.3	.929	.004	.014	------	------	0.4	0.8

It should be borne in mind that the mean *variabilities* of these pyrheliometric observations at different stations are comparable, although the instruments or the methods of reading them may not be standardized to yield comparable *absolute* values. As should be expected (cf. Criteria 7 and 8), wide fluctuations in the observed intensity, A_1, associated with similar fluctuations in the transmission coefficient, a, occur at sea level with the entire mass of absorbing and scattering atmosphere above the station. Thus at Washington the mean variability of the two elements is about ±7 per cent.

[7] The ratio of the probable error to the mean variability is 0.60; to the mean deviation it is 0.85.

On the other hand at the extreme highest stations the variability of A_1 is reduced to ± 0.85 per cent.

It is clear, therefore, that the pure pyrheliometric data show progressively less scatter as we ascend in the atmosphere, and furthermore that the percentage variability of the scatter of a is approximately equal to, and proceeds *pari passu* with that of A_1 as shown by the fidelity with which the data lie along the line, drawn at 45°, joining equal percentages of mean variability of a and A_1.

The inference from the diagram is that at still higher elevations the variations of A_1 would show a proportional decrease. If, however, real solar variation exists, the line joining the variabilities of a and A_1 would, at the limit of the atmosphere, intersect the x-axis at an appreciable distance to the right of the origin. The scatter of a would vanish, and the scatter of A_1, exclusive

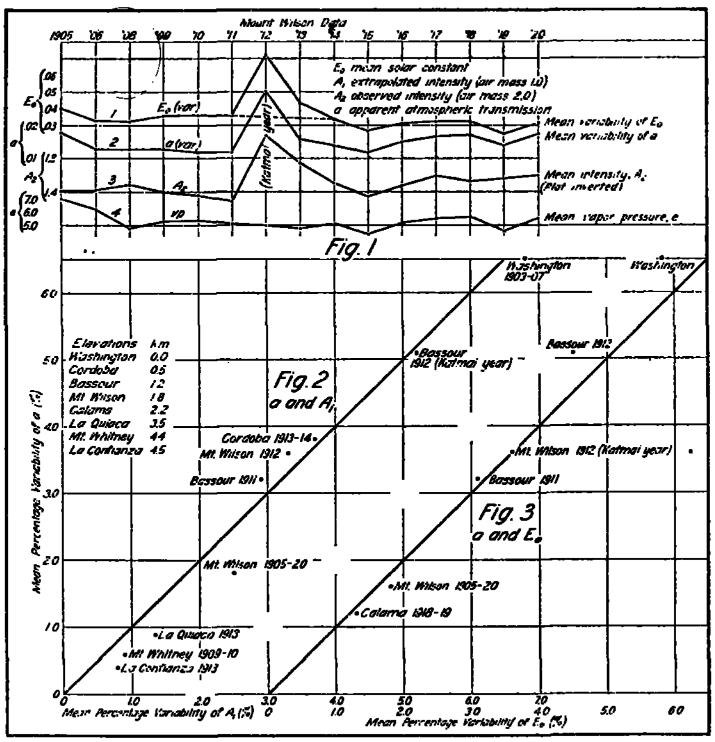

PLATE I.—Yearly values of solar radiation elements at Mount Wilson and mean percentage variability of a, A_1, and E_0, at Smithsonian and Argentine observing stations

of instrumental error, would obviously coincide with that of the real E_0 at this point.

Figure 3 shows a similar progressive decrease in the scatter of E_0 as the elevation of the station increases. The stations are those of the Smithsonian Institution, and the values for E_0 are those made by the long method only, and hence are entirely comparable. The scatter of the values obtained by the pyranometer method at Calama is about half that by the long method. The closeness with which the data lie along the 45° line is remarkable and indicates *correlation in excess of +0.90 between the variations of* a *and* E_0 *at different stations.*

The small value of the scatter of E_0 at Calama, for example, is due entirely to the correspondingly small scatter of the atmospheric transmission coefficient at this point, and hence can not be held to result from any improvement in the method of reduction.

What the observations show, therefore (figs. 2 and 3), is a very rapid diminution of all variations, both of the intensities A_1 and E_0, and of a. If solar variability formed even one-half of the variations of A_1 at Mount Wilson or Calama, the latter should cease to diminish, even though the variations of a vanished altogether. Instead, all the variations diminish *pari passu* almost as if they would vanish at the same time, and one is compelled to interpret those results as fixing the range of possible solar variation, if it exists at all, as only a small fraction of 1 per cent.

Smallness of possible solar variation shown by small scatter of A_1 *at very high altitudes.*—There are data available from four stations, all occupied before 1914. K. Ångström made a series of observations of high merit at Teneriffe in 1896, June 21 to July 3. At Alta Vista, 3,260 meters, observations were taken near midday on eight days and the mean of the maximum intensities was 1.612 cal., with a mean deviation of ± 0.010 cal., or ± 0.62 per cent. At the Peak, 3,700 meters, observations were obtained on two days, the difference between them being 0.008 cal. The 10 observations yield a mean deviation of ± 0.54 per cent. The air mass in no case exceeded 1.01. The mean vapor pressure was 2.8 mm. At Alta Vista the correlation between the observed pressure and the maximum intensity, A_1 is $+0.65$. Applying a correction to the data, A_1 for variation in pressure, the mean deviation is reduced from ± 0.62 per cent to ± 0.47 per cent.

Ångström made eight simultaneous readings with the two instruments he employed there. The means of the two series of readings were identical, and from the mean of the differences between them, ± 0.011 cal., the probable error of a single observation by one instrument is computed to be $\frac{.011 \times .6}{1.61} = \pm 0.41$ per cent. The maximum readings at Alta Vista, corrected for pressure, gave a mean deviation of ± 0.47 per cent. The probable error would be 85 per cent of this, or ± 0.40 per cent, which is about the probable instrumental error.

In 1909 and 1910 Abbot made observations at Mount Whitney, four of which he reduced and published in Vol. III of the Annals. These four days, together with a fifth day of high grade, August 25, 1909, yield a mean deviation of ± 0.47 per cent from the mean of A_1, 1.70 cal. The mean vapor pressure was 2.2 mm.

In August, 1913, A. K. Ångström made observations on eight days at Mount Whitney. Bigelow's determination of A_1 from Ångström's data gave a mean of 1.664 cal., with a mean deviation of ± 0.54 per cent. The mean deviation of the transmission coefficient, a, was ± 0.55 per cent. The smallest air mass ranged from 1.09 to 1.80. The mean vapor pressure was 2.7 mm.

In August and September, 1913, pyrheliometric observations on eight days were made at La Confianza under the direction of Bigelow, and his reductions show a mean value of A_1, 1.740 cal., and a mean deviation of ± 0.59 per cent. Mean vapor pressure was 1.3 mm. The mean deviation of a was ± 0.32 per cent.

These four independent series, at elevations ranging from 3.25 to 4.5 km., yield mean deviations of A_1, ranging from ± 0.47 per cent to ± 0.59 per cent, with an average of ± 0.54 per cent. While the total number of the observations, 31, is not large, yet in view of the noteworthy consistency of the results it seems reasonable to infer that at an elevation of 4.0 km. on days of a high grade of excellence with vapor pressure 2.0 mm. or less, the mean deviation of A_1 will average not greater than ± 0.50 per cent of the mean, and the probable error around ± 0.40 per cent.

Smallness of possible solar variation shown by small scatter of A_1 on days with very low vapor pressure.—(*a*) *at La Quiaca.*[8]—Values of A_1 for 1913 were grouped for definite ranges of vapor pressure and the results are shown in Table 7.

TABLE 7.—*Influence of atmospheric water vapor on values and scatter of A_1 at La Quiaca*

Cases	Range of vapor pressure	Mean variability of A_1		Mean value A_1
	mm.	*cal.*	*per cent*	*cal.*
[1]9	0.4 to 0.7	0.011	0.62	1.780
31	0.4 to 1.2	.017	.95	1.760
26	1.3 to 2.2	.023	1.34	1.720
19	2.3 to 4.7	.029	1.75	1.650
9	4.8 to 6.2	.031	2.00	1.580

[1] These cases are included in the 31 cases with range 0.4 to 1.2.

The relation between A_1 and the vapor pressure is a linear one for values of e less than 5.5 mm. For each decrease of one millimeter in the vapor pressure there is an increase of 0.044 calories in the value of A_1. The value of A_1 corresponding to zero vapor pressure is 1.800 calories. A plot of the above tabular values for the mean vapor pressure and the mean variability of A_1 shows that the relation between these two variables can be represented by a curve whose intersection with the axis at zero vapor pressure corresponds to a mean variability of ± 0.010 calories for A_1, or ± 0.56 per cent of 1.800. The approximate probable error is ± 0.35 per cent.

The La Quiaca data thus fully confirm the results from the four fragmentary series above cited. The probable error of the best observations at air mass 1 with vapor pressure averaging 0.5 mm. is about ± 0.40 per cent.

These high level observations, five independent series, were all made prior to 1914, and give *closely accordant results*, showing that at elevations of 3.5 to 4.5 km. on excellent days with vapor pressure averaging 1.5 mm. the probable error of the extrapolated intensity A_1 is about ± 0.40 per cent.

(*b*) *at Mount Wilson.*—A selection of 53 determinations of A_1 at Mount Wilson during the years 1910 to 1920, exclusive of 1912 and 1913, comprising all days with precipitable water 3.5 mm. or less, and of very good or excellent grade, yielded a mean variability of ± 0.98 per cent, a mean deviation of ± 0.80 per cent and a probable error of about ± 0.65 per cent. The mean variability of E_0 during these years was about ± 1.6 per cent, and the probable error about ± 1.1 per cent, or 0.60 per cent greater than that of A_1 on high grade days. These 53 selected days must be regarded as a random sample, and because of this the probable variation of true solar variation on these days should not differ much from the variation based on all days. This value of the probable error of A_1 on high grade days at Mount Wilson, is consistent with the value, one-third less, derived from stations at elevations of 3.5 to 4.5 km.

(*c*) *at Calama.*—The probable error of the values of A_1 at Calama during the years 1918 and 1919, grouped in accordance with amounts of precipitable water varying by half a millimeter, was ± 0.66 per cent for amounts of precipitable water averaging 1.0 mms., and increased to ± 0.74 per cent for amounts averaging 4.0 mm. During this time the probable error of E_0 was about ± 0.90 per cent. It is probable that the greater prevalence of dust at Calama was partly responsible for the increased amount of scatter there as compared with Mount Wilson.

The source and extent of the variations in A_1.—These may be summarized as (*a*) Solar variations. Small fluctuations are possible but not proved. (*b*) Atmospheric variations, caused by the absorption or scatter by the variable contents, aqueous vapor, ice crystals and dust. The amount of these variations decreases with increasing altitude. (*c*) Errors of observation, nearly constant at all altitudes. These include the error of extrapolation to air mass one, already discussed and the instrumental error. The probable error of a single observation by a copper-disk pyrheliometer is stated by Abbot (Annals Vol. IV, p. 162) to be ± 0.37 per cent, while for the silver-disk instrument the probable error is ± 0.20 per cent. This latter value is doubtless too low for observations under average observing conditions. The Mount Wilson observations were made with a copper-disk instrument, as were the early observations on Mount Whitney. Bigelow's observations in South America were made with a silver-disk instrument. Of the Ångström observations nothing is known beyond the determination of the probable error of the instruments read at Teneriffe in 1896, which, as stated above, was ± 0.41 per cent.

If the probable error of A_1 at 4 km. be regarded as ± 0.40 per cent, and the probable instrumental error be placed at ± 0.30 per cent, it is obvious that after allowance is made for residual atmospheric variations, and error of extrapolation, little is left for solar variation.

CONCLUSIONS

Summarizing the foregoing analyses, it is found (1) that the fluctuations of all day-to-day, and even monthly and yearly mean values of the solar constant are so unmistakably correlated with the transparency of the atmosphere, with its water vapor content and seasonal characteristics that they can not be regarded as even approximate values of real changes in solar intensity.

(2) That whether the comparison is made between measurements of intensities and their scatter, made at widely different times and at stations differing greatly in altitude and therefore in actual air mass, or between intensities, as if at air mass one, and their scatter, at different high-altitude stations where skies are clear and water vapor content nearly zero, we arrive at the one result, that day-to-day and other short-interval fluctuations become progressively smaller, the dryer, clearer, and lesser the air mass through which incoming radiation passes before reaching the measuring instruments.

All these results point to the conclusion that if one could but wholly remove the atmosphere with all its depletions and fluctuations of intensity of transmission, and all instrumental errors, scarcely any variations of radiation intensity would remain. The results are the more satisfactory because the data from whatever source tell the same consistent story, notwithstanding that the observations in some cases are few, made at widely different times, at widely separated stations, and by different observers using entirely different instruments.

If it be conceded that small solar changes amounting to perhaps one to two-tenths of one per cent (measured as a probable error) are possible, it still remains necessary to prove their existence. In the meantime, one can scarcely fail to be impressed with the evidence for the almost absolute constancy of solar radiation.

[8] It should be stated that a careful examination of the La Quiaca data show that a marked discontinuity in the value of A_1 occurred about the middle of May, for some unaccountable reason. The mean values of A_1 and e for the month ending the 16th was 1.687 calories and 1.8 respectively. For the remainder of the month the means were 1.760 and 1.7. Thus with a nearly constant vapor pressure the value of A_1 was higher by 0.07 cal. The two series were regarded as each homogeneous, but in computing the mean variability the discontinuity was allowed for by taking the weighted average of the separate mean variabilities of the two series.